## חAmIBIA UחIVERSITY

OF SCIEПCE AПD TECHПOLOGY
FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: BACHELOR OF ECONOMICS |  |
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| QUALIFICATION CODE: 07BECO | LEVEL: 5 |
| COURSE CODE: MFE512S | COURSE NAME: MATHEMATICS FOR <br> ECONOMISTS 1B |
| SESSION: NOVEMBER 2019 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| FIRST OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| MODERATOR: |  |

## INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. For each question of Section $A$, write $T$ for the true statement and $F$ for the false one. For each question in Section B, write down the letter that corresponds to the correct answer only. For Section C, show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

## ATTACHMENTS

1. Graph paper

THIS QUESTION PAPER CONSISTS OF 5 PAGES (Including this front page)

## SECTION A (True or false Questions)

## QUESTION 1 [15 marks]

State whether each of the following statement is true or false.

### 1.1 A $3 \times 2$ matrix has 2 rows and 3 columns.

$$
x+y-z=0
$$

1.2 The system $2 x+2 z=y$ is homogeneous.
1.3 If $A$ is a $3 \times 4$ matrix and $B$ is a $4 \times 3$ matrix, then the product $B A$ is a $3 \times 3$ matrix.
(1)
1.4 Every square matrix has an inverse.

$$
\text { 1.5 The system } \begin{align*}
& x+2 y=0  \tag{1}\\
& 2 x=y
\end{align*} \text { has only one solution. }
$$

1.6 For any matrix $M, M^{T} M$ is possible.
(1)
1.7 A unit or identity matrix is a matrix such that every entry which is not in the main diagonal is a 0 .
1.8 Two matrices may only be multiplied if they have the same order.
1.9 If matrix $M$ is a singular matrix (has no inverse), then $M=M^{T}$.
1.10 Any matrix of any order may be multiplied by a scalar.
(1)
$1.11\left(\begin{array}{cc}-1 & 2 \\ 2 & -4\end{array}\right)$ is singular.
1.12 Every homogeneous system of equations has a solution.
1.13 Elementary row operations can be used to find the determinant of a non-square
matrix.
1.14 Jacobians are used to test for functional dependence between functions of several
variables.
1.15 Hessian determinants are used to test if a matrix has an inverse.

## SECTION B (Multiple choice Questions)

## Question 2 [10 marks]

Write down the letter that corresponds to the right answer only.
2.1 Which of the following is an example of an upper triangular matrix?
A. $\left(\begin{array}{ll}1 & 0 \\ 2 & 0\end{array}\right)$
B. $\left(\begin{array}{ccc}1 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 3 & 4\end{array}\right)$
C. $\left(\begin{array}{ll}1 & 2 \\ 0 & 0\end{array}\right)$
D. $\left(\begin{array}{ll}1 & 0 \\ 0 & 0 \\ 0 & 2\end{array}\right)$
E. None of these
2.2 Which of the following matrix is singular?
A. $\left(\begin{array}{ll}1 & 0 \\ 2 & 2\end{array}\right)$
B. $\left(\begin{array}{ccc}1 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 3 & 4\end{array}\right)$
C. $\left(\begin{array}{ll}1 & 2 \\ 0 & 0\end{array}\right)$
D. $\left(\begin{array}{ll}1 & 0 \\ 0 & 0 \\ 0 & 2\end{array}\right)$
E. None of these
2.3 Which of the following linear programming problem is a standard maximizing problem?
Maximize $P=3 x-2 y$
A.

$$
\begin{aligned}
& \text { Subject to } 2 x+3 y \leq 6 \\
& \\
& x+2 \leq 1-y \\
& x, y \geq 0
\end{aligned}
$$

Maximize $P=3 x-2 y$
B.
Subject to $2 x+3 y \leq 6$
.

$$
\begin{aligned}
& x-y \leq 1 \\
& x \leq 0, y \geq 0
\end{aligned}
$$

Maximize $P=3 x-2 y$
Minimize $P=3 x-2 y$
Subject to $2 x+3 y \leq 6$
$x-y \leq 1$
$x, y \geq 0$
E. None of these
2.4 The solution for the inequality: $3 \leq 5-2 x \leq 11$ is
A. $1 \leq x \leq-3$
B. $x \geq 1$
C. $-3 \leq x \leq 1$
D. No solution
2.5 Testing the Hessian matrix of the function $f(x, y)=-\frac{1}{2} x^{2}+2 x y+y^{2}+6 x-6 y-10$, at the critical point $(2,1)$ the nature of the critical point is:
A. a local maximum
B. a local minimum
C. a saddle point
D. indeterminate (test fails)

## SECTION C (Structured questions)

Question 3 [75 marks]
3.1 If $A=\left(\begin{array}{c}2 \\ -1 \\ 1\end{array}\right)$ and $B=\left(\begin{array}{lll}1 & -2 & -3\end{array}\right)$ determine the products $A B$ and $B A$ if they exist. (5)
3.2 Use Cramer's rule to solve each of the following systems of linear equations.
$x-y+z=2$
3.2.1 $y+x-z=1+y$ for $y$ only.
$z+y+2=1$
$x+2 y=3$
3.2.2 $\begin{array}{ll} & 3 x+6 y=9\end{array}$

Find the inverse of $A=\left(\begin{array}{lll}1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6\end{array}\right)$ if it exists and use matrix inverse method to solve the
system
$x+2 y+3 z=2$
$2 x+4 y+5 z=3$.
$3 x+5 y+6 z=4$
3.4 In Super Bowl I, on January 15, 1967, the Green Bay Packers defeated the Kansas City Chiefs by a score of 35 to 10 . The total points scored came from a combination of touchdowns, extra-point kicks, and field goals, worth 6, 1, and 3 points, respectively. The numbers of touchdowns and extra-point kicks were equal. There were six times as many touchdowns as field goals. (Source: National Football League)

Set up a system of linear equations to find the numbers of touchdowns, extra-point kicks, and field goals that were scored and solve your system using Gaussian elimination.
3.5 A veterinarian has been asked to prepare a diet for a group of dogs to be used in a nutrition study at a School of Animal Science. It has been stipulated that each serving should be no larger than 8 kgs and must contain at least 29 units of nutrient I and 20 units of nutrient II. He has decided that the diet may be prepared from two brands of dog food: Brand A and Brand B. Each kg of Brand A contains 3 units of nutrient I and 4 units of nutrient II. Each kg of Brand B contains 5 units of nutrient I and 2 units of nutrient II. The cost of Brand $A$ is $N \$ 3$ per kg and $\mathrm{N} \$ 4$ per kg . Determine the number of kgs of each brand of dog food that should be used to meet the given requirements at a minimum cost and determine the minimum cost. Use the graphical method. (15)
3.6 A motor company manufacture and sell cars and motorbikes. The cost of
manufacturing $x$ motorbikes and $y$ cars is given by $C(x, y)=200 x^{2}+200 x y+800 y^{2}$.
Each motorbike is sold for $N \$ 36000-00$ and each car is sold for $N \$ 180000-00$.
3.6.1 Use Gaussian elimination to determine the number of motorbikes and the number of cars that should be manufactured and sold for a maximum profit $\Pi$ and determine the maximum profit $\Pi_{\text {max }}$.
3.6.2 Use the Hessian to confirm that the amounts in 3.6 .1 will produce maximum profit.
3.6.3 Use the Jacobian to test for functional dependence between the cost function and the revenue function.

## *END OF EXAMINATION*

GOODLUCK!!!!!!!!!

